PhD POSITION

In the framework of the research project "Innovative systems and services for transport and production" IDEX/I-SITE CAP 20-25 (Challenge 2) and the LabEx IMobS[3], LIMOS-CNRS laboratory (https://limos.isima.fr/) proposes a 3 year funding (2077 euros brut monthly) for a PhD in Computer Sciences.

Keywords: Algorithmic; Graph Theory; Complexity

The PhD candidate will mainly work in Clermont-Ferrand, One-month stays per year in Tokyo are scheduled and funded with the position.

SUBJECT

Enumeration of Interesting Substructures (EIS)

CONTEXT

When considering an optimization problem, an algorithm that solves the problem is expected to provide only one solution. In the context of enumeration algorithms, for a given problem the algorithm has to list all the solutions. As the number of solutions might be exponential, the classical notion of efficiency of algorithms (namely polynomial time algorithms) has been adapted to capture the same behaviour. An enumeration algorithm is considered efficient, if it manage to list all the solutions in a time polynomially proportional to the size of the input plus the size of all the solutions. In that case the algorithm is said to be output polynomial. One of the main drawback with output polynomial time algorithm is that, the time to produce a first solution might be very long. In order to improve, that, incremental polynomial time algorithm were created. Under this formalism. The time to produce a solution is polynomially bounded by the size of the input and the already found solutions. But this latter notion can be further improved with polynomial delays algorithm where a the time to produce a new solution is only polynomially dependent in the size of the input.

In this project we would like to study the problems consisting in enumerating maximal subgraphs of a graph that fulfills a given property. In some sense, a lot of enumeration problems on graphs can be expressed in that way; for instance a lot of known results exist for some simple families of graphs such as for maximal Independent sets [JPY88], spanning trees (i.e. maximal acyclic subgraphs) [STU97], maximal matchings [U97]. A lot of interesting techniques have been developed for these classes. However for some classes it was shown that the problem turns out to be difficult (namely minimal dominating sets of a graph) [KLMN14] and the minimal di-cuts of directed graphs [KBE08].

OBJECTIVES

This project will study the graph deletion problems and graph completion problems. A deletion problem on a graph G consists in removing a minimal set of edges to G such that the resulting graph fulfills a given property. For instance one can consider the property to be planar, bipartite, or Interval graph... It was shown that finding a minimum deletion is usually NP-hard for any interesting graph property [Y91] whereas finding a minimal deletion is usually polynomial. We will also investigate the vertex deletion problem; in that setting, the goal is to remove a minimal set of vertices in order to obtain a graph that fulfills a given property.

This latter problem in some sense is related to well-known problem in enumeration, namely transversal hypergraph problem. For the vertex deletion problem, the goal is to break all the forbidden subgraphs in the graph. For instance if we want to obtain a cograph, it suffices to remove all the P_4 of the graph (i.e. induced path of length 4). If we want to obtain a minimal deletion, then the result is merely a minimal transversal of all the P_4. In [KLMN14] we manage to prove that listing minimal dominating set is as hard as enumerating transversals of an hypergraph. But, in a series of joint papers with T. Uno we manage to design efficient algorithms for restricted graph classes [KLMNU13, KLMNU15a,KLMNU15b].

The techniques developed in this series of papers could prove to be useful to solve the aforementioned problem, or could be extended to fit our needs.

From the Enumeration point of view this problems received very little attention until recently. From bio-informatics motivations this deletion problem was solved by [CGM+16] for sub-classes of bipartite graphs.

Another problem we want to study, is the completion problem. The completion problem consist in adding a minimal set of edges to the graph such that resulting graph fulfills a given property. When the minimum is asked, the problem is NP-Hard, but for several classes, the minimal completion problem was extensively studied when only one solution is looked for. We can cite the following papers: [CPT15,CT13,HMP08,HM09,HTV05,LMP10] that respectively consider the problem for cographs, Interval graphs, Split graphs, comparability graphs and chordal graphs. The completion problem on the latter class has been extensively studied because of the strong links with treewidth. As for the other problem, the enumeration version, received little attention until recently. From databases motivations Carmelli and her co-authors [CKK17] considered the problem for chordal completions. But a polynomial delay for this class is not known yet.

Both problems are actually related one to another by a complementary relation; for instance if we want to find a maximal bipartite sub-graphs contained in a graph, it is the same as asking to complete the complementary graph into a co-bipartite graph. Some well-known classes has a good property to be self-complementary. For instance Cographs, Permutation graphs, Split graphs are self-complementary classes.

During this project we intend to work on both problems for main graph classes, such as, Chordal graphs, Comparability graphs, Planar and Bipartite graphs... The nature of the expected results is as follows:

- Identify what are the classes that admits output polynomial algorithms.

- Determine some necessary and sufficient conditions that make the problem easy.

- What is, for each class, the underlying structure of a completion (is it an antimatroid, an accessible set...)

- Try to apply some of the results to real world data.

We intend to start our study on easy classes, namely self-complementary classes, where both problems are computationally equivalent one to another. In a first time the targeted classes will be Split graphs and Cographs.

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